# Recent Trends in Nonlinear Partial Differential Equations and Applications

Celebrating Enzo Mitidieri's 60th birthday

May 28–30, 2014

May 28

9.00 Registration

May 28 - Morning session.

Chairman: Filippo Gazzola

9.30-10.20 **Ermanno Lanconelli**: $L^p$-Liouville Theorems for Invariant Evolution Equations

10.30-11.00 Coffee Break

11.00-11.50 **Lucio Boccardo**: Do not worry about elliptic problems with $W^{1,1}$ estimates: sometime it is possible to be happy

12.00-12.50 **Siegfried Carl**: Multi-Valued Elliptic Variational Inequalities: Existence, Comparison, Extremality

May 28 - Afternoon session.

Chairwoman: Patrizia Pucci

14.30-15.20 **Marie-Françoise Bidaut-Véron**: A priori estimates and initial trace for a Hamilton-Jacobi equation with gradient absorption terms

15.30-16.20 **Irena Lasiecka**: Quasilinear dynamics arising in a 3-D fluid structure interactions with moving interface

16.30-17.00 Coffee break

17.00-17.50 **Roberta Musina**: The Navier-Sobolev constant

18.00-18.50 **Alessandro Fonda**: On the higher dimensional Poincaré-Birkhoff theorem for Hamiltonian flows

May 29 - Morning session.

Chairman: Lucio Boccardo

9.15-10.05 **Djairo de Figueiredo**: Sobolev spaces of functions with symmetry and applications to higher order equations

10.15-10.45 Coffee Break

10.45-11.35 **Roberto Triggiani**: Global uniqueness and stability of an inverse problem for the Schrödinger equation on a Riemannian manifold via one boundary measurement

11.45-12.35 **Laurent Véron**: Quasilinear Lane-Emden equations with absorption and measure data

May 29 - Afternoon session.

Chairman: Vicentiu Radulescu

14.15-15.05 **Patrizia Pucci**: Existence of entire solutions

15.15-16.05 **Alberto Farina**: Some new results on entire solutions of an elliptic system arising in phase separation

16.15-16.30 The Rector of the University of Trieste - Professor Maurizio Fermeglia

16.30-17.00 Coffee Break

17.00-17.50 **Vincenzo Vespri**: Pointwise estimates for nonnegative solutions to a class of degenerate/singular parabolic equations

18.00-18.50 NA TMA Editorial Board meeting

21.00- ? Social Dinner

May 30 - Morning session.

Chairman: Ermanno Lanconelli

9.30-10.20 **Julian Lopez-Gomez**: The theorem of characterization of the strong maximum principle

10.30-11.00 Coffee Break

11.00-11.50 **Filippo Gazzola**: A new mathematical explanation of what triggered the catastrophic torsional mode of the Tacoma Narrows Bridge

12.00-12.50 **Guido Sweers**: Two problems for the bilaplace under Dirichlet boundary conditions

May 30 - Afternoon session.

Chairman: Alberto Farina

14.30-15.20 **Vladimir Georgiev**: On continuity of the solution map for the periodic cubic half - wave equation

15.30-16.00 Coffee Break

16.00-16.50 **Ioan Vrabie**: Delay evolution equations with implicit nonlocal initial conditions

17.00-17.50 **Vicentiu Radulescu**: Nonlinear elliptic problems on the Sierpinski gasket

**Marie-Françoise Bidaut-Véron**

Université de Tours, France
veronmf@univ-tours.fr

## A priori estimates and initial trace for a Hamilton-Jacobi equation with gradient absorption terms

May 28 - 14.30

**Abstract.** Here we consider the nonnegative solutions of the parabolic Hamilton-Jacobi equation

$$u_t - \Delta u + |\nabla u|^q = 0$$

in $\Omega \times (0, T)$ where $\Omega = \mathbb{R}^N$ or $\Omega$ is a bounded domain of $\mathbb{R}^N$, $q > 0$.

We give new a priori local or global estimates for solutions, without conditions as $|x| \to \infty$ or $x \to \partial\Omega$, and corresponding existence results with an initial data measure $u_0 \in \mathcal{M}^+(\Omega)$.

We also study the existence of an initial trace. We show that all the solutions admit a trace as a Borel measure $(S, u_0)$ : there exist a set $\mathcal{S} \subset \Omega$ such that $\mathcal{R} = \Omega \backslash \mathcal{S}$ is open, and a (possibly unbounded) measure $u_0 \in \mathcal{M}^+(\mathcal{R})$, such that

$$\lim_{t \to 0} \int_{\mathcal{R}} u(.,t)\psi = \int_{\mathcal{R}} \psi d\mu_0, \qquad \forall \psi \in C_c^0(\mathcal{R}),$$

$$\lim_{t \to 0} \int_{\mathcal{U} \cap \mathcal{S}} u(.,t)dx = \infty, \qquad \forall \mathcal{U} \text{ open} \subset \Omega, \text{s.th. } \mathcal{U} \cap \mathcal{S} \neq \emptyset.$$

We give more generally existence results of solutions with such a trace, according to assumptions on $u_0$, and give their behaviour as $|x| \to \infty$. In particular we construct a solution with trace $(\mathbb{R}^{N+}, 0)$. When $q \leqq 1$, we show that $\mathcal{S}$ is empty.

## Lucio Boccardo

Università "La Sapienza", Roma, Italia


# Do not worry about elliptic problems with $W^{1,1}$ estimates: sometime it is possible to be happy

May 28 - 11.00


**Abstract.** Let $\Omega$ be a bounded open set in $\mathbb{R}^N$, $N \geq 2$. The simplest example of nonlinear boundary value problem is the Dirichlet problem for the $p$–Laplace operator, with $1 < p < N$, $0 < \alpha \leq a(x) \leq \beta$,

$$\begin{cases} A(u) = -\mathrm{div}\big(a(x)|\nabla u|^{p-2}\nabla u\big) = f(x), & \text{in } \Omega; \\ u = 0, & \text{on } \partial\Omega; \end{cases}$$

so that the growth of the differential operator is $p-1$. The classical theory of nonlinear elliptic equations states that $\wp$ is the natural functional spaces framework to find weak solutions, if the function $f$ belongs to the dual space of $\wp$.

The existence of $W_0^{1,p}(\Omega)$ solutions fails if the right hand side is a function $f \in L^m(\Omega)$, $m \geq 1$, which does not belong to the dual space of $W_0^{1,p}(\Omega)$: it is possible to find distributional solutions (joint papers with T. Gallouet) in function spaces "larger" than $W_0^{1,p}(\Omega)$, but contained in $W_0^{1,1}(\Omega)$.

Then we proved the following existence results (again joint paper with T. Gallouet) of $W_0^{1,1}(\Omega)$ distributional solutions.

THEOREM 1 *Let $f \in L^m(\Omega)$, $m = \frac{N}{N(p-1)+1}$, $1 < p < 2 - \frac{1}{N}$. Then there exists a distributional solution $u \in W_0^{1,1}(\Omega)$.*

THEOREM 2 *Assume $p = 2 - \frac{1}{N}$ and*

$$\int_\Omega |f| \log(1 + |f|) < \infty.$$

*Then there exists a distributional solution $u \in W_0^{1,1}(\Omega)$.*

Then I will present a work in progress (with Rita Cirmi) concerning the existence of solutions in $W_0^{1,1}(\Omega)$ of unilateral problems.

## Siegfried Carl

Martin-Luther-Universität Halle-Wittenberg, Halle, Germany

### Multi-Valued Elliptic Variational Inequalities: Existence, Comparison, Extremality

May 28 - 12.00

**Abstract.** We consider elliptic variational inequalities governed by nonlinear (Leray-Lions) elliptic operators which include lower order upper semicontinuous multi-valued terms. An important special case of an upper semicontinuous multi-valued function is given by Clarke's gradient of some locally Lipschitz function. We are going to provide an analytical framework in terms of appropriately defined sub-supersolutions for elliptic variational inequalities that will allow us to prove the existence of extremal solutions within some given ordered interval. This approach is useful in the study of problems where, e.g., coercivity fails to hold. Further, using that new tool of sub-supersolution, we are going to show that a number of variational-hemivariational inequalities are indeed equivalent to some subclasses of multi-valued variational inequalities considered here. This fills a gap in the literature where both problems are investigated separately.

## Djairo de Figueiredo

Universidade Estadual de Campinas, Brasil

### Sobolev spaces of functions with symmetry and applications to higher order equations

May 29 - 9.15

**Abstract.** We present some imbeddings of Sobolev spaces with different types of symmetry into Lebesgue spaces with weights. We then apply to some PDE's.

---

## Alberto Farina

Université de Picardie Jules Verne
LAMFA, CNRS UMR 7352, Amiens, France

### Some new results on entire solutions of an elliptic system arising in phase separation

May 29 - 15.15

**Abstract.** We study the positive solutions of the elliptic system

$$\begin{cases} \Delta u = uv^2 & \text{in } \mathbb{R}^N \\ \Delta v = vu^2 & \text{in } \mathbb{R}^N, \end{cases}$$

where $N \geq 2$.
We present some new results concerning the monotonicity and the one-dimensional symmetry of solutions with algebraic growth at infinity.

---

## Alessandro Fonda

Università di Trieste

### On the higher dimensional Poincaré-Birkhoff theorem for Hamiltonian flows

May 28 - 18.00

**Abstract.** We propose some extensions to higher dimensions of the Poincaré - Birkhoff Theorem for Poincaré time-maps of Hamiltonian systems. Our results can be applied to pendulum-type systems and weakly-coupled superlinear systems.

**Filippo Gazzola**

Politecnico di Milano

# A new mathematical explanation of what triggered the catastrophic torsional mode of the Tacoma Narrows Bridge

May 30 - 11.00

**Abstract.** The spectacular collapse of the Tacoma Narrows Bridge has attracted the attention of engineers, physicists, and mathematicians in the last 74 years. There have been many attempts to explain this amazing event, but none is universally accepted. It is however well established that the main culprit was the unexpected appearance of torsional oscillations. We suggest a mathematical model for the study of the dynamical behavior of suspension bridges which provides a new explanation for the appearance of torsional oscillations during the Tacoma collapse. We show that internal resonances, which depend on the bridge structure only, are the source of torsional oscillations. Some quantitative explanations, matching the parameters of the Tacoma Bridge, will also be given.

**Vladimir Georgiev**

University of Pisa, Largo Bruno Pontecorvo 5, 56100 Pisa, Italy
`georgiev@dm.unipi.it`

# On continuity of the solution map for the periodic cubic half - wave equation

May 30 - 14.30

**Abstract.** We consider the Cauchy problems associated with the following two nonlinear equations

$$(\mathrm{i}\partial_t - |D_x|)u = \sigma |u|^2 u \text{ for } t \geq 0 \ , \tag{1}$$

where $\sigma = \pm 1$. We shall assume that $u(t, x)$ is $2\pi-$ periodic in $x$. If we have solutions $u(t, x) \in C([0, T]; H^s(0, 2\pi))$, with $s > 1/2$, then the equation have at least two conservation laws

$$\|u(t)\|_{L^2(0,2\pi)} = const$$

and

$$\frac{1}{2}\||D|^{1/2}u(t)\|^2_{L^2} + \frac{\sigma}{4}\|u(t)\|^4_{L^4} = const. \tag{2}$$

Definition 1 *The problem* (1) *is well - posed in* $H^s(0, 2\pi)$ *with* $s \in (0, 1)$ *if for any* $R > 0$ *one can find* $T = T(R) > 0$ *so that for any data* $u(0) = f \in H^s$ *with* $\|f\|_{H^s} \leq R$ *one can define unique solution* $u(t, x) \in C([0, T]; H^s)$ *so that the solution map*

$$f \in B(R) = \{g \in H^s; \|g\|_{H^s} \leq R\} \to u(t, x) \in C([0, T]; H^s)$$

*is continuous.*

A stronger property is the uniform continuity of the solution map.

Our main result is the following.

Theorem 1 *For any* $s \in (1/3, 1/2)$ *the Cauchy problem for*

$$(\mathrm{i}\partial_t - |D_x|)u = |u|^2 u \textit{ for } t \geq 0 \ , \tag{3}$$

*can not have uniformly continuous solution map in* $H^s$.

## Ermanno Lanconelli

Università di Bologna

### $L^p$-Liouville Theorems for Invariant Evolution Equations

May 28 - 9.30

**Abstract.** Some $L^p$-Liouville theorems for several classes of evolution equations will be presented. The involved operators are left invariant with respect to Lie group composition laws in $R^{n+1}$. Results for both solutions and sub-solutions will be given.

## Irena Lasiecka

University of Memphis, Memphis, TN 38152-3370

### Quasilinear dynamics arising in a 3-D fluid structure interactions with moving interface

May 28 - 15.30

**Abstract.** Equations of fluid structure interactions are described by Navier Stokes equations coupled to a dynamic system of elasticity. The coupling is on a free boundary interface between the two regions. The interface is moving with the velocity of the flow. The resulting model is a quasilinear system with parabolic-hyperbolic coupling acting on a moving boundary. One of the main features and difficulty in handling the problem is a mismatch of regularity between parabolic and hyperbolic dynamics. The existence and uniqueness of smooth local solutions has been established by D. Coutand and S. Shkoller *Arch. Rational Mechanics and Analysis* in 2005. Other local wellposedness results with a decreased amount of necessary smoothness have been proved in a series of papers by I. Kukavica, A. Tuffaha and M. Ziane. The main contribution of the present paper is global existence of smooth solutions. This is accomplished by exploiting a natural damping occurring at the interface along with a propagation of maximal parabolic regularity enjoyed by one component of the system.

This work is joint with M. Ignatova (Stanford University), I. Kukavica (University of Southern California, Los Angeles) and A. Tuffaha (The Petroleum Institute, Abu Dhabi, UAE)

**Julian Lopez-Gomez**

Universidad Complutense Madrid, Spain

## The theorem of characterization of the strong maximum principle

May 30 - 9.30

**Abstract.** The pioneering work of E. Mitidieri in collaboration with D. de Figueiredo on the maximum principle for cooperative systems facilitated tremendously the theorem of characterization of the strong maximum principle in the setting of cooperative systems. After some years it became apparent the great importance of these ideas not only for cooperative systems but for single operators.

**Roberta Musina**

Dipartimento di Matematica ed Informatica
Università di Udine, Italia

## The Navier-Sobolev constant

May 28 - 17.00

**Abstract.** I will report on two joint papers with Alexander I. Nazarov, St. Petersburg Department of Steklov Institute. We deal with fractional Sobolev spaces of any real order $m > 0$ over a bounded and smooth domain $\Omega \subset \mathbb{R}^n$.

Assume $n > 2m$ and put $2^*_m := \frac{2m}{n-2m}$. Let $\langle (-\Delta)^m_D \cdot, \cdot \rangle$ be the quadratic form associated to the "Dirichlet" fractional Laplacian $(-\Delta)^m_D$, that is defined via Fourier transform. The Hardy-Littlewood-Sobolev inequality implies that the infimum

$$S^D_m(\Omega) := \inf_{\substack{u \in C^\infty_c(\Omega) \\ u \neq 0}} \frac{\langle (-\Delta)^m_D u, u \rangle}{\|u\|^2_{2^*_m}}$$

is positive. It is easy to prove that $S^D_m(\Omega) = S_m(\mathbb{R}^n)$, and in particular $S^D_m(\Omega)$ does not depend on $\Omega$.

Next, we use spectral decomposition to define the "Navier" fractional Laplacian $(-\Delta)^m_N$ as the $m^{\text{th}}$-power of the standard Laplacian. The naturally related function space $H^m_N(\Omega)$ is the domain of the quadratic form $\langle (-\Delta)^m_N u, u \rangle$.

We generalize a result obtained for integer orders $m$ by Van der Vorst (1996), Ge (2003) and Gazzola-Grunau-Sweers (2010) about the best constant

$$S^N_m(\Omega) := \inf_{\substack{u \in H^m_N(\Omega) \\ u \neq 0}} \frac{\langle (-\Delta)^m_N u, u \rangle}{\|u\|^2_{2^*_m}}.$$

**Theorem.** *Equality* $S^N_m(\Omega) = S_m(\mathbb{R}^n)$ *holds, for any real* $m > 0$.

Preliminary and related results of independent interest are discussed as well.

---

[MN1] R. Musina and A. I. Nazarov, On fractional Laplacians, Comm. Partial Differential Equations, online first (2014). DOI: 10.1080/03605302.2013.864304.

[MN1] R. Musina and A. I. Nazarov, The Navier-Sobolev constant, in progress.

## Patrizia Pucci

Università di Perugia

### Existence of entire solutions

May 29 - 14.15

**Abstract.** The talk deals with the existence of entire solutions of a quasilinear equation in $\mathbb{R}^N$, which involves a general variable exponent elliptic operator **A** of the $p(x)$-Laplacian type in divergence form and two main nonlinearities of growth $q = q(x)$ and $r = r(x)$. The results we present extend the previous work in several directions. We first weaken the condition $\max\{2, p\} < q < \min\{r, p^*\}$ to the simpler request that $1 << q << r$. We also ask milder assumptions on the coefficients of the nonlinearities, as well as a very weak ellipticity condition on **A**. The results we present are new even in the case of constant exponents and even in the semilinear case $p \equiv 2$.

## Vicentiu Radulescu

Mathematics Institute of the Romanian Academy
Bucharest, Romania

### Nonlinear elliptic problems on the Sierpinski gasket

May 30 - 17.00

**Abstract.** We develop some recent results concerning the qualitative analysis of solutions to some nonlinear elliptic problems on fractal domains. Our analysis includes the case of nonlinear terms with oscillatory behaviour, either at the origin or at infinity. The approach combines variational arguments with the geometrical properties of the Sierpinski gasket.

This talk reports on some recent results with G. Molica Bisci.

**Guido Sweers**

Universität zu Köln

# Two problems for the bilaplace under Dirichlet boundary conditions

May 30 - 12.00

**Abstract.** In two dimensions the corresponding boundary value problem is known as the Kirchhoff-Love model for the clamped plate:

$$\begin{cases} \Delta^2 u = f & \text{in } \Omega, \\ u = |\nabla u| = 0 & \text{on } \partial\Omega. \end{cases} \tag{1}$$

• The first item I would like to discuss is based on a joint work Colette de Coster and Serge Nicaise. Finding numerical approximations of a solution to (1) by piecewise linear finite elements one searches for a stationary point of a discretized version of $J : H_0^1(\Omega) \times H^1(\Omega) \to \mathbb{R}$, defined by

$$J(u,w) := \int_\Omega \left( \nabla u \cdot \nabla w - fu - \tfrac{1}{2} w^2 \right) dx.$$

This approach by Monk, which is based on a related variational approach of Ciarlet-Raviart, indeed gives the solution of (1) when the domain is smooth. What happens when the domain has corners?

• The second item concerns a question of Svetlana Mayboroda: *If $f = 1$ in (1), does it hold that $u \geq 0$ for $\Omega$ an arbitrary smooth domain?* Such questions have a long history. Boggio and Hadamard conjectured around 1900 that, at least for convex domains, $f \geq 0$ implies $u \geq 0$. Duffin in 1946 was the first to find a counterexample. In 1952 Duffin also found a counterexample for the claim that the corresponding first eigenfunction is of one sign. So most expectations concerning positivity for fourth order problems turned out to be false; also for the present question. Hans-Christoph Grunau and I were able to find a domain where the solution of (1) with $f = 1$ changes sign.

**Roberto Triggiani**

University of Memphis, Memphis, TN 38152-3370

## Global uniqueness and stability of an inverse problem for the Schrodinger equation on a Riemannian manifold via one boundary measurement

May 29 - 10.45

**Abstract.** We consider a mixed problem for the Schrodinger equation on a finite dimensional Riemannian manifold with magnetic and electric potential coefficients and non-homogeneous Dirichlet boundary term. The goal is the nonlinear problem of the recovery of the electric potential coefficient by means of only one boundary measurement on an explicitly identified portion of the boundary. We obtain global uniqueness of the recovery and Lipschitz stability of the recovery exclusively in terms of the data of the problem. Two key ingredients are: (i) a Carleman estimate for the Schrodinger equation on a Riemannian manifold (Triggiani-Xu, 2008); (ii) optimal interior and boundary regularity of the direct mixed problem (Lasiecka-Triggiani, 1991). This is joint work with Zhifei Zhang.

## Vincenzo Vespri

Dipartimento di Matematica ed Informatica Ulisse Dini
Universitá degli Studi di Firenze
Viale Morgagni, 67/a I-50134 Firenze (ITALY)
vincenzo.vespri@unifi.it

### Pointwise estimates for nonnegative solutions to a class of degenerate/singular parabolic equations

May 29 - 17.00

**Abstract.** Let us consider the following homogeneous quasilinear parabolic equations whose prototypes are the $p$-Laplacian ($\frac{2N}{N+1} < p < \infty$) and the Porous medium equation ($(\frac{N-2}{N})_+ < m < \infty$).

$$u_t = \operatorname{div} A(x, t, u, Du), \quad (x, t) \in \mathbb{R}^N \times [0, +\infty), \tag{1}$$

where the functions $A := (A_1, \dots, A_N)$ are assumed to be only measurable in $(x, t) \in \mathbb{R}^N \times [0, +\infty)$, continuous with respect to $u$ and $Du$ for almost all $(x, t)$.

By using recent results obtained in collaboration with Bögelein, Calahorrano, Düzgün, Fino, Piro Vernier and Ragnedda we are able to give some pointwise estimates from above and from below starting from the value of the solution attained in a point. We apply these results to give sharp estimates to the fundamental solution of such class of equations

## Ioan Vrabie

"Al. I. Cuza" University of Iaşi, Romania
ivrabie@uaic.ro

### Delay evolution equations with implicit nonlocal initial conditions

May 30 - 16.00

**Abstract.** Recent results concerning abstract nonlinear delay evolution equations subjected to implicit nonlocal conditions are discussed. The class of nonlocal conditions considered is general enough to include: periodic, anti-periodic as well as mean conditions and so it encompasses a large variety of nonlinear PDE's models in Physics, Chemistry, Biology, Population Dynamics and Meteorology.

*Acknowledgements.* Supported by the Romanian National Authority for Scientific Research, CNCS–UEFISCDI, project number PN-II-ID-PCE-2011-3-0052.

# Laurent Véron

Université François Rabelais, Tours 37200, France


## Quasilinear Lane-Emden equations with absorption and measure data

May 29 - 11.45

Joint work with Nguyen Quoc Hung


**Abstract.** Let $\Omega \subset \mathbb{R}^N$ $(N \geq 2)$ be a bounded domain with $C^2$ boundary and $g : \Omega \times \mathbb{R}_+ \to \mathbb{R}_+$ be a Caratheodory function, nondecreasing in the second variable such that $g(x, 0) = 0$.

We investigate several questions associated to the following equation

$$\begin{aligned} -\Delta_p u + g(x, u) = \mu \qquad & \text{in } \Omega \\ u = 0 \qquad & \text{on } \partial\Omega \end{aligned}$$

where $\Delta_p u = div\left(|\nabla u|^{p-2}\nabla u\right)$, $p > 1$ and $\mu \in \mathfrak{M}^b(\Omega)$. Typical cases

$$g(x, u) = |x|^{-\beta}|u|^{q-1}u \;\text{ and }\; g(x, u) = \mathrm{sgn(u)}(\mathrm{e}^{\tau|\mathrm{u}|^\lambda} - 1).$$

We prove that solutions exist provided the measure satisfies some absolute continuity property with respect to some appropriate Lorentz-Bessel capacity. Our results are the counterpart of the ones obtained by Phuc and Verbitsky concerning the equations with source reaction

$$\begin{aligned} -\Delta_p u = g(x, u) + \mu \qquad & \text{in } \Omega \\ u = 0 \qquad & \text{on } \partial\Omega \end{aligned}$$